Recent analysis of the leverage effect for the main index on the Warsaw Stock Exchange

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Abstract

In this paper we examine four asymmetric GARCH type models and one (basic) symmetric GARCH model. In particular, the analysis is limited to GARCH(1,1). We analysed daily logarithmic returns of WIG (the main index of the Warsaw Stock Exchange). The aim of the analysis was to check whether there is a so-called "leverage effect" on the Warsaw Stock Exchange in the recent period (since the accession of the European Union). In particular, if volatility of returns from the index is differently affected by the changes of the index itself. We have found that the analysis weakly confirms the hypothesis of the existence of such an effect, but the results show that a continuation of the current research is still reasonable.

Keywords

Asymmetric GARCH models, leverage effect, Poland, Warsaw Stock Exchange

JEL Classification

C22, G11, G17

Introduction

Time series appearing in finance tend to present leptokurticity and variance clustering. It means that the empirical distribution of a variable under analysis does not resemble the normal distribution. Returns presented on the histogram present higher peaks, meaning that the observations are densely concentrated around the average value. On the other hand, the tails are still significant. Roughly speaking, the distribution is like a "squeezed" normal density plot.

Variance clustering means that periods when the volatility is high are sequenced by period of lower volatility. The typical example of such behaviour are the financial markets and stock prices. In certain periods, investors' interest in specific stocks is small, hence the price fluctuations are lower in these periods. On the other hand, under the increased interest, the price volatility increases.

Nelson (1991) noted that generally there is a negative correlation between the current interest returns and future volatility of rates of return for various assets. Hence, it is reasonable to consider similar analysis for the Polish stock exchange.

Literature review

Modelling of time series, for which there exists the mentioned phenomena is very popular within the GARCH methodology (Bollerslev, 1986). In this paper it is assumed that the variable x_t follows GARCH(p,q) process, if

(1) $x_t = \mu + e_t$,

where $e_t = u_t \sqrt{h_t}$ and u_t follows the standard normal distribution (i.e., with mean 0 and variance 1). Index t denotes the time. Moreover,

(2) $h_t = \omega + \alpha_1 \cdot (e_{t-1})^2 + \ldots + \alpha_p \cdot (e_{t-p})^2 + \beta_1 \cdot h_{t-1} + \ldots + \beta_q \cdot h_{t-q}$.

In most cases, it turns out that GARCH (1,1) is sufficient (Hansen and Lunde, 2005). Then, Eq. (1) remains unchanged, and Eq. (2) takes the following form

(3) $h_t = \omega + \alpha_1 \cdot (e_{t-1})^2 + \beta_1 \cdot h_{t-1}$.

However, Nelson (1991) criticized the concept of GARCH in this form. He noted that generally there is a negative correlation between the current interest returns and future volatility of rates of return for various assets. Moreover, during the estimation of a GARCH model usually one finds inconsistent results with the theoretical limitations. It is required that $\alpha_1 , \ldots , \alpha_p$ and $\beta_1 , \ldots , \beta_q$ are positive. The first mentioned drawback is sometimes called "leverage effect".

It consists of the fact that asset prices usually increase when volatility decreases, and vice versa: when the price falls – volatility increases. In the paper of Nelson (1991) a modification of the standard (basic) GARCH model was proposed. This gave rise to the so-called asymmetric GARCH models family (Tsay, 2002; Zivot, 2009).

For example, in T-GARCH(1,1) Eq. (3) is replaced by the following one

(4) $\sqrt{h_t} = \omega + \alpha_1 \cdot \sqrt{h_{t-1}} \cdot (| z_{t-1}| - \eta_1 \cdot z_{t-1}) + \beta_1 \cdot \sqrt{h_{t-1}}$,

where $z_t = e_t / \sqrt{h_t}$.

In GJR-GARCH(1,1) Eq. (3) is replaced by the following one

(5) $\ln (h_t) = \omega + \alpha_1 \cdot (e_{t-1})^2 + \gamma_1 \cdot I_{t-1} \cdot (e_{t-1})^2 + \beta_1 \cdot \ln (h_{t-1})$,

where $I_{t-1} = 1$ if $e_{t-1} < 0$ and $I_{t-1} = 0$ in other cases.

In E-GARCH(1,1) Eq. (3) is replaced by the following one

(6) $\ln (h_t) = \omega + \alpha_1 \cdot z_{t-1} + \gamma_1 \cdot (| z_{t-1}| - E|z_{t-1}|) + \beta_1 \cdot h_{t-1}$.

Finally, in APARCH(1,1) Eq. (3) is replaced by the following one

(7) $(h_t)^\delta = \omega + \alpha_1 \cdot (| e_{t-1} | - \gamma_1 \cdot e_{t-1})^\delta + \beta_1 \cdot (h_{t-1})^\delta$.

Sandoval (2006) found that from a practical point of view asymmetric GARCH models should not be favoured over classic GARCH model. In contrast, Harrison and Moore (2012) found that for countries of Central and Eastern Europe asymmetric GARCH models significantly better describe the behaviour of the stock market indices than the symmetric (basic) GARCH model. Similar findings for the countries of South-eastern Europe were presented by Okičić (2014).

In case of Poland, Kobus and Pierzykowaski (2006) found that the occurrence of "leverage effect" on the Warsaw Stock Exchange is really present only for MIDWIG index. In particular, the main WIG index does not present any asymmetric leverage effects. Their study was based on daily closing prices from the period between 5 January 1998 and 29 May 2006. It seems therefore interesting to consider the analysis of a more recent period (including, for example, the recent global financial crisis), but limited to time after the Polish accession of the European Union.

For the Polish market the work of Fiszeder (2009) and references cited therein are also very important ones.

Data description

The daily closing levels of WIG index were taken from http://stooq.pl/q/g/?s=wig. The considered period begins on 4 May 2004 and ends on 25 August 2015. The closing level of WIG is denoted by WIG_t , where t stands for the time index. As a result, 2837 observations were collected. Basing on these data the daily logarithmic rate of returns were computed and denoted by x_t, i.e.

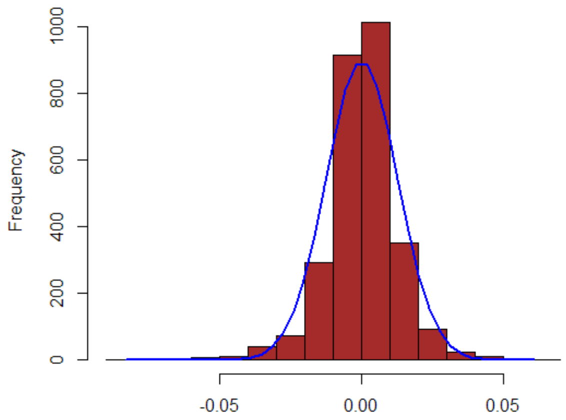
(8) $x_t = \ln (WIG_t / WIG_{t-1})$.

Tab. 1: Descriptive statistics for the variable x

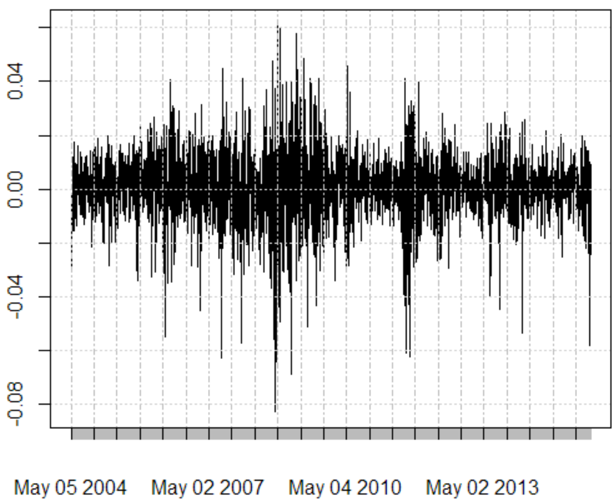
no of obs.	mean	stand. dev.	min	max	skewness	kurtosis
2836	0	0.01	-0.08	0.06	-0.51	1.03.1994

It can be seen (Tab. 1) that the mean is close to zero. From Pic. 1 it can be seen that x does not follow the normal distribution. The leptokurticity is clearly present. Yet, this is confirmed by the very small p-value for the Jarque Bera test at 5% significance level. Moreover, from Pic. 2 the variance clustering is clearly seen. A very high volatility is especially seen around 2008.

Pic. 1: Histogram of the variable x



Pic. 2: Time series plot for the variable x



The construction of GARCH models for the variable x is reasonable. Indeed, the very small p-value for the LM test of ARCH effects (Lagrange multiplier) confirms that there are significant ARCH effect at 5% significance level.

Finally, it seems from Pic. 2 that the values of x oscillate around zero. In other words, no significant time trend is seen. Indeed, the ADF test (augmented Dickey-Fuller) confirms the stationarity of the variable x (ADF statistic is -12.824 and p-value is 0.01) at 5% significance level. The means does not seem to be changing with time and the current values do not depend on the past values.

Estimations and diagnostic

The results of the estimation and diagnostic are presented in Tab. 2. The computation were done in R with “rugarch” package (R Core Team, 2015; Ghalanos, 2014).

Tab. 2: Estimations and diagnostic results

	GARCH(1,1)	E-GARCH(1,1)	T-GARCH(1,1)	GJR-GARCH(1,1)	APARCH(1,1)
estimates					
μ		0.000445	0.000454	0.000389	0.000384
ω	0.000001*	-0.141253	0.000187	0.000002*	0.000000*
α_1	0.074185	-0.055029	0.081828	0.042093	0.056907
β_1	0.918048	0.983765	0.921401	0.916070	0.911526
γ_1		0.156074		0.057977	0.160226
η_1			0.382048		
δ					2.567477
diagnostic (p-values)					
LB	0.8073	0.7741	0.9755	0.6443	0.5963
ARCH LM	0.5774	0.9248	0.8598	0.8050	0.7561

* denotes statistical non-significance at 5% level

The estimates of T-GARCH(1,1) suggest that there exists the asymmetric leverage effect. In other words, a negative shock increases volatility much more than a positive shock. The same conclusion can be drawn from estimations of GJR-GARCH(1,1), E-GARCH(1,1) and APARCH(1,1).

It should be noticed that all estimated model are stationary. This is because $\alpha_1 + \beta_1 < 1$. Actually, for E-GARCH(1,1) it is enough to check that $\beta_1 < 1$, and therefore, the negativity of α_1 is not problematic (Zivot, 2009).

In Tab. 3 Akaike information criteria are presented for all estimated models. The model with the lowest Akaike information criterion (AIC) is preferred. The idea is to compare both the complication of a model (i.e., number of variables) and the model's quality. In other words, whether higher complication of a model brings a significant benefit in model's quality.

Tab. 3: Akaike information criteria for estimated models

model	AIC	relative probability

GARCH(1,1)	-6.1708	0.994117
E-GARCH(1,1)	-6.1826	1.000000
T-GARCH(1,1)	-6.1821	0.999750
GJR-GARCH(1,1)	-6.1800	0.998701
APARCH(1,1)	-6.1753	0.996357

It can be seen (Tab. 3) that the most preferred is E-GARCH(1,1) model. It can also be mentioned that the parameter δ for APARCH model is close to 2. Notice, that for $\delta = 2$ APARCH reduced to GJR-GARCH model.

Relative probabilities can also be computed (Tab. 3). Let AIC_{min} denote the AIC for the model with the smallest Akaike information criterion and let AIC_i denote the AIC of the i-th model. The relative probability of the i-th model (Burnham and Anderson, 2002) is then defined as the number $\exp[0.5(AIC_{min} - AIC_i)]$ and can be interpreted as the relative probability that the i-th model minimises the information loss (i.e., is preferred).

Now, it can be seen that there is not much evidence to support the hypothesis that E-GARCH(1,1) is really better than simple GARCH(1,1). For example, GARCH(1,1) is 0.994117 times as probable as E-GARCH(1,1) to minimise the information loss. Similarly, relative probabilities for all other models are very high. Therefore, the support for the hypothesis that asymmetric models are significantly better than simple GARCH(1,1) is very weak.

High p-values for Ljung-Box test for autocorrelation of residuals (Tab. 2) suggests that in case of all estimated models there is no significant autocorrelation of squared residuals. Similarly, there is no evidence for remaining ARCH effects (Lagrange multiplier test), because of high p-values (Tab. 2) for all estimated models.

Finally, in Tab. 4 mean squared errors for all estimated models are reported. It can be seen that mean square error is quite similar for all estimated modes. As a result, the forecasts obtained with the help of all models have quite similar quality. However, the smallest mean square error is for APARCH(1,1) model.

As a result, the selection based on minimising MSE leads to the different conclusion than Akaike information criterion.

Tab. 4: MSE for estimated models

model	**MSE**

GARCH(1,1)	0.0001597712
E-GARCH(1,1)	0.0001597486
T-GARCH(1,1)	0.0001597523
GJR-GARCH(1,1)	0.0001597297
APARCH(1,1)	0

It is important to notice that all models passed diagnostic tests and lead to similar conclusion on asymmetric leverage effect. Although, a few parameters occurred not to be statistically significant (Tab. 2), it is not problematic. They can be just simply assumed equal to zero, without significant change in the methodology. The numerical outcomes are also quite similar. Yet, due to mentioned objections the reported study needs further, more detailed, continuation of research.

Conclusions

Four asymmetric GARCH(1,1)-type models were estimated and diagnosed: E-GARCH(1,1), T-GARCH(1,1), GJR-GARCH(1,1) and APARCH(1,1). Also the symmetric GARCH(1,1) model was estimated and diagnosed. For all models the Akaike information criterion was computed. This criterion preferred E-GARCH(1,1) model. Indeed, all models passed diagnostic tests. All asymmetric models confirmed the initial hypothesis that there is the leverage effect on the Warsaw Stock Exchange. Nevertheless, the evidence is quite weak. On the other hand, the smallest mean squared error was observed for APARCH(1,1). This research was based on daily logarithmic returns from the main index (WIG) between 2004 and 2015. Still, a continuation of this research seems reasonable, because basing on relative probabilities the evidence of the existence of the asymmetric leverage effect is very weak.

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