The Effect of Stock Market Overvaluation on Merton's Model

Jakub TABAČEK

Abstract: This paper tests the effect of equity market misvaluation of publicly traded companies on their respective default probabilities. We use a natural experiment of Dot Com bubble that allows us to isolate the misvaluation effect of stock market. Merton's credit risk model is used for estimating individual default probabilities. Probit regression is estimated to compare the model's predictive ability during Internet bubble with the rest of the sample while controlling for industry and economic conditions. Evidence suggests that market overvaluation causes Merton's model to underestimate credit risk.

Keywords: Merton Models; Corporate Failure; Implied Default Probabilities

JEL: G12, G13

1 Introduction

Credit risk measurement and assessment are indispensable for daily functioning of financial institutions. It is critically important for solvency of banks and insurance companies that are at the centre of developed economy. With the advance of computers and larger databases of defaults, traditional human risk assessors had been supplanted by quantitative risk models. This practice has been also encouraged by regulatory requirements of Basel[1]. Complexity and scale of portfolios of these institutions necessitates quantitative models for effective risk assessment. Although there have been numerous efforts in development of risk models recent crisis of 2008 has raised awareness of their deficiencies. This paper will address a common modelling drawback that we believe has not been formally recognised in the academic literature.

There are two major classes of models, each using different approach. *Reduced-form models* simulate default as Poisson-distributed surprise event that arrives with certain intensity λ. *Structural models* assume default occurs when firm's asset value process reaches a default threshold usually represented by firm's liabilities. Both classes of models have advantages and disadvantages, however structural models are very intuitive and thus easily explained to senior management or wider audience beyond academics. For this reason, they have been well received by credit risk practitioners and regulators (BIS, 2001)

Although there are countless variations of structural models, for the aims of this paper we decided to use the original Merton's model (1974). Merton was first who applied the ground-breaking discovery of Black and Sholes' (1973) option pricing model. Modigliani and Miller (1954) earlier showed that equity and debt are can be expressed as options on the underlying firm assets. Merton applies this idea to pricing risky corporate debt through estimating the probability of default (PD). In this model, equity value and volatility are arguably the most influential parameters driving the PD. (Ramaswamy and Sundaresan, 1993). The PD is decreasing as firms assets rise relative to liabilities and increasing with volatility. Using Black-Scholes formula, unobserved value of assets is derived from observed value of equity and debt. Debt here is relatively static and usually follows deterministic path[3]. Valuation of firm's equity is assumed to be observed through the stock market.

Whether or not stock market is an efficient way to vale equity has been a prime theme in the history of financial economics. The main debate is focused on the weak and the semi-strong form of Efficient Market Hypothesis (EMH). Where the interest of former is testing various form of random walks, the latter looks at performance of professional investors. The central notion revolves around ability or inability of investors to systematically outperform the "market". Markets are thus said to be efficient if economic agents cannot earn excess profits (Summers, 1986). However, such state of efficiency by itself does not force markets to properly vale firm's equity and risk thereof. Prices can include a random element that reflects expectations, fads, uncertainty or second order expectations (Diba and Grossman, 1988). Event when equity price diverges from fundamental value is a natural phenomenon facilitated by market frictions and noise (Fama, 1965). According to Black (1989) noise is present in all aspects of the economy and economic activity. Therefore accumulation of noise can yield significant departures of market prices from assets' intrinsic value. There have been several periods in the economic history when booming markets were quickly followed by equally magnificent crashes. Dot Com bubble of late 90s is perhaps the best example of such boom-bust process (Griffin et al. 2009).

In this paper we use this extreme event in order to examine the effect of market overvaluation on the PDs of publicly traded companies. We estimate PDs for large number of heterogeneous companies across different industries for individual years 1996-2004. Then we compare Internet companies' PDs of the year 2000 with the rest of the sample. A pooled cross-section probit regression is deployed for this purpose. The dependent variable is the state of an individual company at the end of a given year which is default or no default. Independent variables are PD of the respective company and a dummy interaction variable for Internet group*year 2000. The regression coefficient of this dummy interaction is the key result of this paper. It determines whether bubble has any effect on the Merton's model. The economic environment, type of firm or level of equity market should not affect estimates of PDs in theory[4]. This underlines our null hypothesis. The Merton PD should be sufficient estimator that already includes all this information from stock market. However, we include controls for year 2000 and for software GICS[5] sub group to add validity to our results. Due to this inclusion any possible deviation relative to average model performance is conditioned on macroeconomic environment (during year 2000) that affect all the companies equally. In addition, conditioning on software industry controls for any possible bias of PD due to specific nature of technology companies. Therefore, our results are not dependent on any other assumptions of Merton's model except the market equity valuation that is subject of our enquiry. Assumptions regarding nature of companies' liabilities and assets, interest rates, capital structure, diffusion process, or bankruptcy negotiations made by the Merton model do not confound our results as there is no reason to assume that Internet companies that are used as a treated group would significantly deviate from the control group represented by the rest of the sample. Experimental design of this study allows for a statistically robust inference. The empirical evidence suggests that elevated market equity prices bias PD estimates of Merton's model.

1.1 Testing Hypothesis

Now we will explain the null and the alternative hypotheses and also state them symbolically. The null hypothesis (H_0) is defined as follows: Sock market bubble has no effect on the Merton's model estimates of 1 year PD. Given the proper estimation of model's parameters, Merton's predictive ability is not dependent on the price of company's shares relative to fundamentals. This is because stock valuation reflects the company's intrinsic value. The true risk of the company's assets will be accounted for through the volatility of the stock prices of the respective company. Therefore information contained in the equity market is well suited to model firms' credit risk effectively by Merton model. Consequently, predictive ability of Merton for Internet companies for the year 2000 should not significantly differ from average predictive ability of Merton for other time periods or other companies. This is after conditioning for the economic environment of 2000 and specifics of IT and software industry.

The purpose of conditioning on control variables is due to Merton's model making other assumptions beyond market valuation. These are described in the literature review section of this paper. It is quite likely that all internet companies PDs would be estimated with systematic error due to nature of the industry (not captured by Merton). It is also likely that all the PDs for the year 2000 will be estimated with error due to business cycle or perhaps interest rate change that Merton's model does not account for. These factors can bias our results and tests. To prevent this bias, we include these two (or more if needed) variables in our probit regression. This will purge the effect (coefficient) of the internet companies in year 2000 from the effect of the year 2000 itself and the internet industry itself. This way we can argue that any anomaly of Merton's model performance during the Dot Com bubble (proxied by Internet 2000) is purely due to the effect of the bubble alone. The choice of probit regression, variables and the setup of the testing procedure are explained in detail in the methodology section.

<u>The alternative hypothesis</u> (H_a) is that null hypothesis is not true. This is the same as negation of the arguments describing the null hypothesis. Therefore under the alternative, stock market bubble has an effect on the performance of the Merton's model. The β_2 coefficient in the true probit model would be positive. The regression analog $\hat{\beta}_2$ would be positive and of course significantly different from zero.

<u>Symbolically</u>:

True model

$$Y = \beta_0 + \beta_1(PD) + \beta_2(Internet2000) + \beta_3(dIND) + \beta_4(d2000) + \sum_{n=5}^{N} \beta_n(ctrl_n)$$

Regression model

$$\mathrm{Pr}obit: Y_i = \hat{\beta}_0 + \hat{\beta}_1(PD_i) + \hat{\beta}_2(Internet2000_i) + \hat{\beta}_3(dIND_i) + \hat{\beta}_4(d2000_i) + \sum_{n=5}^{N} \hat{\beta}_n(ctrl_{n,i}) + \varepsilon_i$$

Hypotheses

$$H_0 : \beta_2 \leq 0$$
$$H_a : \beta_2 > 0$$

Internet2000 is a dummy interaction variable which equals 1 if the Merton model is estimating PD for the year 2000 and for an internet company. I order to purge the $\hat{\beta}_2$ coefficient from any possible bias dIND and d2000 variables are added. These are also dummy variables that equal 1 if a given firm is in the internet industry and PD is estimated for the year 2000 respectively. This is a standard practice in econometrics to condition on anything that is correlated to our variable of interest the dependent variable. This way we can make a valid inference on the $\hat{\beta}_2$ coefficient. This is similar to Gauss-Markov exogeneity assumption in the OLS. Detailed explanation is provided in the Methodology section. The presence of additional control variables at the end of the probit model is due to alternative specifications that we test as well. We show that after controlling for different sets of variables our coefficient is unaffected. This is elaborated in the results.

Based on the relationship of the stock market and Merton's model we expect the null hypothesis to be rejected. However, only data and statistical testing will have the final word in this inquiry. It is reasonable to assume that market overvaluation would directly translate to the abnormally low PD for some internet companies that defaulted within 1 year. On the other side of the argument, it is also possible that these technology companies have high variances that made their PDs large enough. Internet companies may be overvalued but the risk contained in their volatile assets and traded equity can easily offset the effect of market overvaluation in the Merton's model. This idea is parallel to option pricing where the price of an option depends on the strike price (relative to current price of the underlying) as well as the volatility. Technology and Internet companies' assets are often intangible and their value can be high on one day, but low on the next. They can be easily misvalued due to the difficulty to value intangibles and company prospects. Such can also lead to high volatility. If the volatility is high, the risk of default is high as well. This way the Merton's model could provide sound PDs even for the internet companies during Dot Com period. In that case we would fail to reject the null hypothesis.

2 Methodology

This section describes method for estimating PD and subsequent tests. The choice of statistical methods was mostly dictated by the data constraints. The data on public company defaults is limited and statistically tractable bubbles do not occur often. First, model of PD is introduced with explanation of the main processes. Second, probabilities of default in 1 year are estimated for period during dot com bubble and period not affected by the bubble. Third, actual and estimated probabilities of default are collected and probit is estimated for implied PDs and actual default outcomes. The testing will focus on the β_2 coefficient which will tell us the effect of our bubble proxy in equation (1).

2.1 Estimation of PD

In this section we outline the method of estimating Merton's PD. Complete statistical treatment of modelling would distract the reader from the main aim of this paper. Therefore the scope is limited to explanation of the fundamental ideas behind the model. For more details see for example Bilecki and Rutkowski (2002).

There are three major components for the estimation of PD. These are value of assets, which measure the company's prospects and incorporate relevant information about the industry and the economy; asset risk, which is represented by the volatility of assets; and leverage, which represents company's debt. As asset values are not observable, we use BS contingent claim analysis. This is why structural models are also called option-theoretic or contingent-claim models. What we do observe is the price of options represented by debt and equity (D, E). Now we would like to obtain the underlying asset parameters, for this we have to walk backwards.

Market value of assets is assumed to follow a stochastic process where z is the standard Brownian motion defined on the probability space $(\Omega, \mathscr{F}, \mathbb{P})$

$$dV_A = \mu V_A dt + \sigma_A V_A dz,$$

assets are assumed to follow a lognormal distribution with constant drift μ and variance σ^2,

$$\ln V_A^T \sim N(\ln V_A^t + (\mu_A - \sigma^2/2)(T-t), \sigma_A^2(T-t)).$$

If L is the value of debt which is assumed to be maturing at time T then the V_A and V_E are related by:

$$V_E = V_A N(d_1) - e^{-r_t T} L N(d_2),$$

$$d_1 = \frac{\ln\left(\frac{V_A}{L}\right) + \left(r_t + \frac{\sigma_A^2}{2}\right)T}{\sigma\sqrt{T}},\ d_2 = d_1 - \sigma_A\sqrt{T},$$

r_t is the risk free rate, V_E^t is the market value of equity.

This result comes from BS option pricing formula and is used to obtain the parameters of asset behaviour from its derivatives. In order to arrive at the value of the yet unknown parameters V_A^t, σ_A we need to solve a system of two equations and two unknowns that considers asset and equity vales and volatilities[9]. As already shown in BS formula:

$$V_E = V_A N(d_1) - e^{-r_t T} L N(d_2).$$

Moreover, from Itô's Lemma, we can write the equity process

$$dV_E = \mu_E V_E dt + \sigma_E V_E dz$$

as

$$dV_E = \frac{\partial V_E}{\partial V_A} dV_A + \frac{\partial V_E}{\partial t} dt + \frac{1}{2}\sigma_E^2 V_A^2 \frac{\partial^2 V_E}{\partial V_A^2}(dV_A)^2 \ldots$$

$$= (\frac{1}{2}\sigma_A^2 V_A^2 \frac{\partial^2 V_E}{\partial V_A^2} + \mu_A V_A \frac{\partial V_E}{\partial V_A} + \frac{\partial V_E}{\partial t})dt + \sigma_A V_A \frac{\partial V_E}{\partial V_A} dz$$

Comparing the diffusion terms of dV_E we can write

$$V_E \sigma_E = V_A \sigma_A \frac{\partial V_E}{\partial V_A}.$$

From BS formula we know that equity delta $\frac{\partial V_E}{\partial V_A}$ corresponds to $N(d_1) > 0$. Thus we obtain a ready relationship linking equity and asset volatilities.

$$V_E \sigma_E = V_A \sigma_A N(d_1)$$

Now we deploy numerical routines in VBA for every i in our sample to solve the system we just derived above:

$$V_E = V_A N(d_1) - e^{-r_t T} L N(d_2)$$

$$V_E \sigma_E = V_A \sigma_A N(d_1)$$

After obtaining the V_A^t, σ_A, we can model the PD as the probability that at point T the value of assets will exceed a default point (DP). Merton originally defines the DP as the total value of liabilities. In our case we use a slight modification by using default point which is a function of liabilities. According to KMV's[10]empirical findings, company defaults when it reaches this default pint (DP) rather than its total liabilities This technically mean that firm does not have enough resources to pay creditors and can be ordered by the court into bankruptcy. Now that we know L, V_A^t, σ_A^2, μ_A determining PD is very intuitive. It can be imagined as probability of X falling below x.

$$PD = \Phi\left[\frac{z - E[X]}{\sigma_X}\right]$$

Substituting, PD is the probability that a log-normally distributed random variable V_A^t falls below a threshold DP.

$$PD = \Phi\left[\frac{\ln DP - \ln V_A^t - (\mu_A - \sigma_A^2 / 2)(T - t)}{\sigma\sqrt{T - t}}\right]$$

$$= \Phi\left[\frac{\ln(DP/V_A^t) - (\mu_A - \sigma_A^2/2)(T-t)}{\sigma\sqrt{T-t}}\right].$$

Figure 1 below depicts the process of asset value hitting a DP barrier at time T with some probability PD.

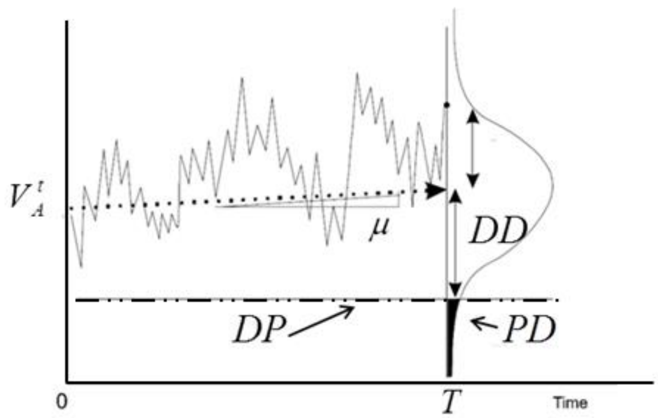

Figure 1

2.2 Tests

Performance of credit risk model can be tested using a range of methods including comparing the number of predicted and actual defaults in a sample of companies, or similarly comparing implied and actual ratings, credit scores, credit spreads or CDS spreads. As the companies that we use for our model analysis do not have traded bonds, CDS or external ratings, we only use the information of their default or non-default at time T , $T \in N[1996;2004]$. For the purpose of testing the null hypothesis we will analyse if there is any statistical deviations in implied PDs from regular model performance. We use a pooled cross-section probit regression in order to include different time periods and industries. This is necessary as we defined the bubble proxy variable as interaction of specific industry and time. The regression is specified as follows:

$$\Pr obit: Y_i = \hat{\beta}_0 + \hat{\beta}_1(PD_i) + \hat{\beta}_2(Internet2000_i) + \hat{\beta}_3(dIND_i) + \hat{\beta}_4(d2000_i) + \sum_{n=5}^{N} \hat{\beta}_n(ctrl_{n,i}) + \varepsilon_i$$

The dependent variable is the outcome of default $Y, Y \in \{0,1\}$. This is always recorded at the end of a given year T regardless if the exact month of the default. Explanatory variables are:

a) our Merton's model implied $PD, PD \in R^{+}\langle 0;1\rangle$;
b) a time dummy that captures any change in Y due to time shift which serves as a control variable for macroeconomic effects on Y;
c) an industry dummy that captures effects present in the industry of Internet companies that controls for any systematic difference in Y due to nature of the firm not accounted by the model;
d) a bubble proxy is specified as a dummy interaction variable of year 2000 and Internet subindustry;
e) other control variables will be used beyond the industry and year dummy to purge $\hat{\beta}_2$ from capturing any other effect besides the stock market overvaluation.

The probit regression is a special functional form of OLS which allows us to model dichotomous outcomes such as default, non-default. This is obtained by taking a function of $G(.)$ of the right hand side of the regression, where $G(.)$ limits the RHS to lie between [0,1]. The functional from of $G(.)$ in probit is the standard normal distribution.

$G(.)$ is of the form $G(x_i;\beta) = F(x_i'\beta)$ with

$$F(w) = \Phi(w) = \int_{-\infty}^{w} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}t^2\right\} dt$$

In general probit describes the probability of $y_i = 1$, $P\{y_i = 1 | x_i\} = G(x_i;\beta)$. This means that probability of $y_i = 1$ depends on the vector x_i containing individual characteristics. The functional form makes the interpretation of the coefficients difficult. From

$$\frac{\partial \Phi(x_i'\beta)}{\partial x_{ik}} = \phi(x_i'\beta)\beta_k$$

we can see that partial effect of x_k depends on β_k as well as on other regressors. However, we can still easily interpret the sign of the partial effect which is the same as the sign of β_k as $\phi(.)$ is nonnegative on its whole domain. For the purpose of our hypothesis test, the sign and statistical significance are the results we are interested in.

Estimation

We can obtain the β_k parameters from using maximum likelihood estimation. The likelihood contribution of observation i with $y_i = 1$ is given by $P\{y_i = 1|x_i\}$ as a function of the unknown parameters β_k, and similarly for $y_i = 0$. The likelihood function of the entire sample is given by:

$$L(\beta) = \prod_{i=1}^{N} P\{y_i = 1|x_i;\beta\}^{y_i} P\{y_i = 0|x_i;\beta\}^{1-y_i}$$

After log-linearizing and taking the first derivative we obtain the following FOC:

$$\frac{\partial \log L(\beta)}{\partial \beta} = \sum_{i=1}^{N} \left[\frac{y_i - F(x_i'\beta)}{F(x_i'\beta)(1-F(x_i'\beta))} f(x_i'\beta) \right] x_i = 0$$

Interestingly the FOC says that the explanatory variable should be orthogonal to the them in the square brackets which is called generalized residual. This is equivalent to Gauss-Markov exogeneity assumption of OLS and is needed to obtain consistent parameters in MLE. Similarly to OLS $E(\varepsilon|\mathbf{x}) = 0$, there must be no linear relationship between the generalised residual and the regressors in our MLE. For this purpose we include control variables that may be correlated with Y and the bubble proxy. MLE of our probit model are computed in STATA.

2.3 Data

For PD estimation we use a sample of US defaults provided by Prof. Gi Kim. We use random selection of companies in years 1996-2004 that represent wide range of industries across the US economy. For each year a sample of defaulters and non-defaulters are selected for which default variable is marked 0 or 1 depending on the default occurrence in given year. Data for calibration is gathered strictly prior to estimating PD that takes place at the end of the year preceding the estimation period $\tau = (T - t)$. For example data for company that defaulted at some time during the year 1998 is gathered from information prior to 1st of January 1998. This is as close as possible to 31st December of 1997 in order to use the most accurate values for calibration.

Firm equity value V_E^t is calculated as the product of the last price on December (T-1) and total shares outstanding at that date.

Volatility of equity is calculated from daily stock price. First prices of the last 12 months January- December of the year (T-1) are collected. Prices provided by CRSP are in their raw form as reported by the exchange. In order to adjust these prices for corporate actions such as stock splits, revere stock splits, mergers, spinoffs, buybacks, etc. we (were fortunate to) obtain cumulative adjustment factors from WRDS matching the prices that allow us to adjust stock prices. Afterwards, daily log returns are calculated and used to obtain sample standard deviation which represents the daily volatility. To get an annualized volatility we take product of the square toot of the trading days and daily volatility.

Risk free interest rate for BS formula is estimated as an average of the overnight Libor rates for period $\tau - 1 = ((T-1) - (t-1))$.

Asset growth rate (μ_i) is approximated from the equity behaviour implied by CAPM. First equity beta of company i β_i is estimated as

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)},$$

then using r_f from LIBOR and r_m from S&P 500 returns r_i of an equity i we use CAPM formula:

$$\mu_i = r_f + \beta_i (R_m).$$

Company total liabilities (L) are obtained from Compustat database of quarterly fundamentals. In order to maximize the sample we used any fundamentals reported during last 6 months of the calendar year preceding the modelled year so L_t =(T-(1,1.5)). So if the company didn't produce financial statements during December (T-1) then we went back and used any statements that the company produced up to 6m prior to that date. For some companies only annual fundamental were available. In such case soonest data available could be for February (T-1) or then next February (T). However this would be either into the PD estimation period or otherwise almost 2y before T which, given the dynamic nature of L, is not accurate information. Such cases were discarded from the sample. Discarding process is orthogonal to any of the parameters in this work other than data availability itself.

Current liabilities were obtained from Compustat in similar fashion. We denote it as *curL*. We also use long term liabilities which are by definition *L-curL=longL.*

Default Point (DP) is calculated using KMV method that replaces L with default threshold DP. KMV studied large number of defaults and found that companies usually default when their assets reach threshold $curL + \frac{1}{2} longL$.

Time, $\tau = (T - t)$ is in Merton's model assumed to be 1.

In addition I use Global Industry Classification Standard (GICS) to identify sector, industry group, industry, and subindustry. These variables were developed by S&P as part of the joint initiative with MSCI Barra to establish a global standard for categorizing companies into sectors and industries. GICS was developed in response to the global financial community's need for one complete, consistent set of global sector and industry definitions. I use this data co define Internet software and services sub-Industry and Internet software and services Industry. Please refer to the GICS transcript in appendix for details.

3 Results

A) PD Estimation

For the purpose of testing the null hypothesis we have estimated 5679 company PDs. The total number of defaults in the sample is 499. The average PD was 4.74 % with standard deviation of 14.17 %. The model implied and actual default outcome has a correlation of 29.98 %. A common statistic that has been adopted in credit risk testing is receiver operating characteristics (ROC) .

The ROC ratio is a measure of accuracy where each observation of default is paired with non-default and compares the PD of each. It gives one point if the company did not default is rated as the better credit, one half point if the two are rated the same and zero if non-default is rated lower than default. This test uses all observations and there is no data mining. A ratio of 100% is the best and 50% is equal to random allocation of PD, which would mean that our PD estimates are bad. Our whole sample has a ROC ratio of 74.37 %. According to Van Deventer and Imai (2003) a ROC ratio around 75 is in the ‘pretty good’ category. For comparison, a state of the art EDF measure presented in MKMV promotional material had a ROC of 86% (Bohn et al., 2005). Figure 2 below depicts our full sample ROC curve.

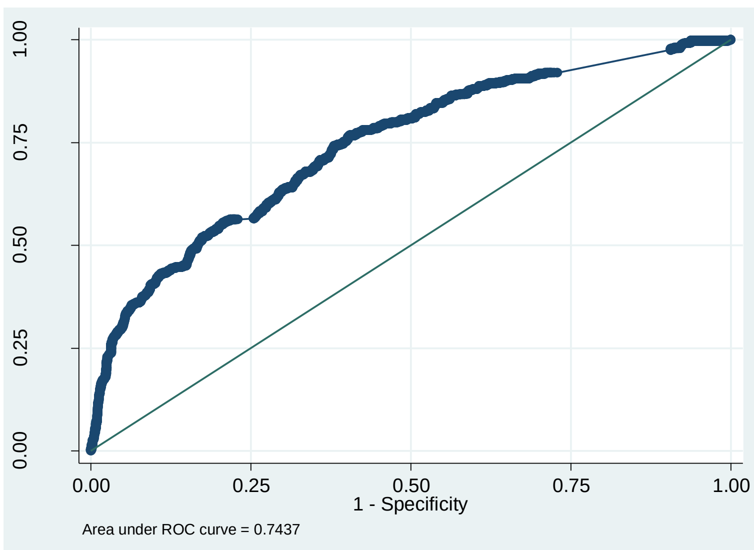

Figure 2

All the industry groups characterised by GICS are represented in the sample. Table 1 below displays the summary statistics across industry groups for the Merton model estimates. Each group has a reasonable ROC ratio so there is no reason to believe that nature of industry would bias the PD estimates. The number of companies in each group is representative of US listed firms during 1996-2004

		GICS Sectors		
Sector name	GICS number	Number of observations	Relative frequency	ROC

Energy	10	292	0.05142	0.717
Materials	15	432	0.07607	0.594
Industrials	20	1035	0.18225	0.725
Consumer discretionary	25	962	0.16940	0.682
Consumer Staples	30	512	0.09016	0.653
Health care	35	718	0.12643	0.786
Financials	40	270	0.04754	0.746
Information Technology	45	1001	0.17626	0.843
Telecommunication Services	50	90	0.01585	0.636
Utilities	55	362	0.06374	0.849

Table 1

Table 2 below describes the summary statistics across years. The ROC ratios suggest Merton's model performance seems to be unaffected by the time period. The number of defaults in each year is representative of the US economy. The reason for generally low number of defaults is due to low default frequencies of listed company defaults in the US as well as data availability for those that did default.

		Year		
Year	Number of defaults	Number of observations	Relative frequency	ROC
1996	24	501	0.0882	0.726
1997	28	524	0.0923	0.727
1998	51	573	0.1009	0.653
1999	20	649	0.1143	0.732
2000	71	692	0.1219	0.785
2001	123	761	0.1340	0.772
2002	69	663	0.1167	0.769
2003	61	687	0.1210	0.773
2004	22	629	0.1108	0.733

Table2

Figure 3 below shows actual and model forecasted number of defaults. The model forecast number is a product of an average PD for a given year and the sample size of given year estimates. We can see that model predictions are correlated to the actual. It seems that the

Merton’s model is underestimating the risk on average. The model is generally better at ranking than in forecasting the absolute level of PD. This systematic under-prediction of risk should not affect the result of our hypothesis test. Allowing for intercept and by including a control variable for the year 2000 logit will capture this systematic deviation. We can think of no reason why the general under-estimation or risk should be specific to our variable of interest representing the bubble, other than the bubble itself.

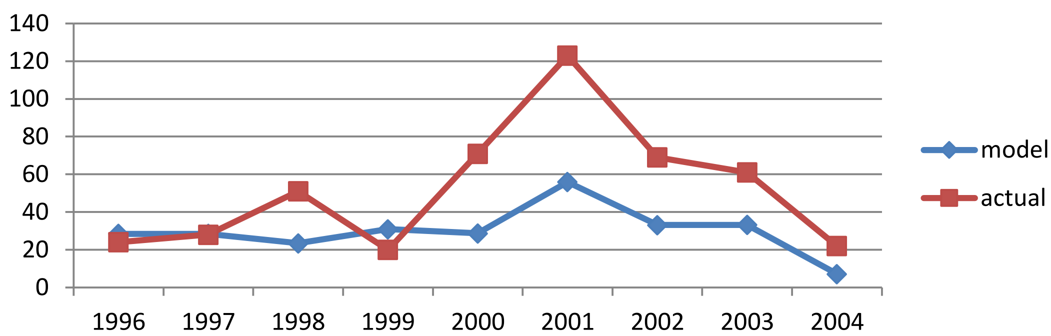

Figure 3

A) *Hypothesis test*

On the graph of Internet firm Caredata that defaulted during the year 2000, we can see that the price of common stock was not exhibiting much concern over default. The PD of this company is 2.15%. The stock market implied value of assets was 256.98% of book values at the end of year 1999. Merton’s model considers this company to be relatively safe and forecasts PD lower than many companies that survived the year 2000. Our sample includes 10 internet companies that defaulted and 15 that did not during the year2000. The average PD of these firms was 0.73% which is too low given that almost half of them defaulted. Now, we will test if the PD estimates are impacted by equity bubble compared to PDs that were not impacted. The Internet companies in the whole sample have a ROC of 80.11% which suggest that model can reasonably well estimate PD for internet companies in general.

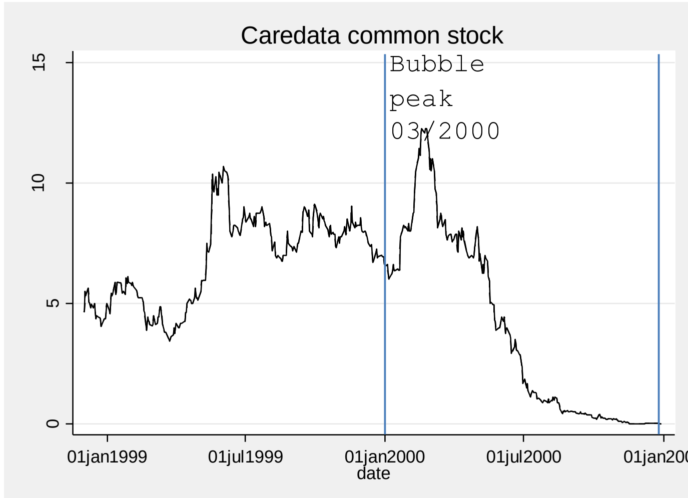

Figure 4

The probit regression results in table 3 below tell us the sign and significance of the bubble captured by Internet2000 proxy.

Table 3

Variable	Coef.	Std. Err.	z	P>|z|
PD	2.1618	0.1275	16.95	0.000
Internet2000	1.8394	0.8649	2.13	0.033
Sector(45)	0.0476	0.1036	0.72	0.474
Industry(4510)	-0.6818	0.2459	-2.84	0.005
y2000	0.1223	0.0751	1.60	0.103
constant	-1.4537	0.3090	-50.19	0.000
pseudo Rsq	0.0969	LR statistic		327.62
		Prob > chi2		0.000

The results suggest that the stock price bubble is associated with significant positive impact on actual default outcome at 5% significance level. The probit model explain 9.69% of the variation in observed defaults with pseudo R^2 being significant at 1% significance level. The coefficient is relatively large and positive. This points to inability of Merton's model to generate sufficiently large default probabilities prior to Dot Com debacle. We argue that this proxy is exogenous to the nature of the Internet firms. The result is conditioned on both Information technology GICS (45) sector and Software and services GICS (4510) industry[11]. The year 2000 is included to isolate our proxy from any macroeconomic bias. Based on this evidence we reject the null hypothesis.

Furthermore, we have added a numerous additional control variables after which the coefficient on Internt2000 is at the same level of standard errors and even greater magnitude of bubble impact.

Table 4

Variable	Coef.	Std. Err.	z	P>|z|
PD	2.1062	0.1320	15.95	0.000
Internet2000	1.3020	0.5839	2.23	0.026
Sector(45)	0.4659	0.1801	2.59	0.010
Industry(4510)	0.3534	0.1240	2.85	0.004
y2000	0.4938	0.1268	3.89	0.000
Sector(10)	0.4692	0.1995	2.35	0.019
Sector(15)	0.4775	0.1894	2.52	0.012
Sector (20)	0.5915	0.1717	3.45	0.001
Sector (25)	1.0457	0.1686	6.20	0.000
Sector (30)	0.3356	0.1911	1.76	0.079
Sector (35)	0.4731	0.1785	2.65	0.008
Sector (40)	-0.1556	0.2550	-0.61	0.542
Sector (50)	1.6172	0.2126	7.61	0.000
y1996	-0.0044	0.1456	-0.03	0.976
y1997	0.0452	0.1417	0.32	0.750
y1998	0.4340	0.1288	3.37	0.001
y1999	0.3459	0.1294	2.67	0.008
y2001	0.7134	0.1195	5.97	0.000
y2002	0.4828	0.1252	3.85	0.000
y2003	0.3790	0.1274	2.97	0.003
y2000*Sector (45)	-0.0865	0.3063	-0.28	0.778
constant	-2.5441	0.1896	-13.42	0.000
pseudo Rsq	0.1674	LR statistic		565.88
		Prob > chi2		0.000

In addition, we estimated the same model with book vales instead of market values for the Internet companies of the year 2000. The resulting PDs had the same market implied volatility of asset but the level of assets was based on accounting estimate. Under assumption that book values are fair and not contaminated by bubble we expect the bubble proxy to turn insignificant.

Table 5 below shows that indeed the Internt2000 variable is not significant at 5 % significance level.

Table 5

Variable	Coef.	Std. Err.	z	P>|z|
PD	2.2029	0.1281	17.20	0.000
Internet2000	0.9904	0.6895	1.28	0.101
Sector(45)	-0.1659	0.0830	3.00	0.048
Industry(4510)	0.3558	0.1169	-2.84	0.005
y2000	0.1163	0.0728	1.60	0.110
constant	-1.5430	0.3080	-50.18	0.000
pseudo Rsq	0.0950	LR statistic		320.02
		Prob > chi2		0.000

3.1 Discussion and Limitations

Here we would like to argue that our results are orthogonal to any other assumption made by Merton model other that assumption about equity valuation. We walk through most common assumptions criticised and amended by research that succeeds Merton (1974)

Taxes: Taxation has been shown to affect both corporate and investors' behaviour (Berk et al., 2014). Some structural models such as Leland (1994) consider the role taxation. We abstract from taxation as to all studied companies being in the US the taxes are fairly similar. In addition we have conditioned for IT and software industry, therefore Internet subindustry taxation practices and capital structures are not likely to affect results.

Debt dynamics and capital structure: Company's debt usually increases as it nears default. The reason for this comes from the structure of payoffs. Decrease in financial condition of company can be solved through rising funds in form of additional equity or debt. Equity holders are in control of the company. As equity represents a call option on the firm's assets, equity holders will try to maximize upside potential by raising more debt. In case the company is not able to turn around, equity holders lose their investment just as it would be had they not borrowed. However, if debt financing helps the company to become profitable, the equity holders benefit. On the contrary, the payoff structure of debtholders provides them with no benefit for increasing the probability of up-state. Simple Merton's model assumes static capital structure. Despite, there is no reason for capital structure of Internet companies at year 2000 to behave differently from other companies in their industry that were also facing default.

Stochastic interest rates: Merton model assumes interest rates to stay fixed through the estimation period. Van Deventer and Imai (2003) argue that interest rates changes are correlated with defaults such as during Savings and Loan crisis. Shimko et al. (1993) extent Merton's model to allow for stochastic interest rates following Vasicek's mean-reverting term structure. In our study, changes of interest rates do not impact the effect of equity price after we control for all the PD deviations due to specific conditions of the time.

Time of default: It can be argued that all the internet stocks defaulted at a specific time e.g. in April when the bubble burst and are this not showing equity assumption but rather the assumption regarding the timing of the default.

Similarly we find no reason why should other assumptions regarding nature of assets (tradability, diffusion process) or type of liabilities (covenants, coupons, maturity, floating rates, etc.) contaminate our findings.

Furthermore, we would also like to drive attention to some efforts that have already have been made in recognition of cyclical effects on structural models. Industry used models such as KMV have long argued that EDF is a sufficient measure of default probability (Crosbie, Bohn. 2002). Stein(2004) proves that additional factors such as price to earnings ratio is a significant factor in addition to EDF. This is due to market mispricing companies' equity given fundamentals. Eventually KMV decided to create through the cycle EDF measure (Moody's Analytics, 20011). Academics have such as Jarow (2001) have also become cautious and decided to explicitly account for stock market bubble in the reduced form models.

We suggest that both cyclical effects and bubble effects should be included implicitly in the structural models that use market data. Another way to help tackle the market valuation errors is to including economic variables in addition to PD forming hybrid models.

Implications of ignoring stock market overvaluation are potentially damaging for the financial sector. Structural models used for capital structure decisions may underestimate the risk of firm's debt and may lead to greater debt issuance by corporations during stock market boom. Similarly banks may understate the risk of their portfolios and lend excessively. Investors may be also deceived to invest in risky corporate debt that has booming equity, which makes the debt look relatively safe. Coalescence of these agents' decisions based on structural models may lead to exaggeration of credit cycles. Where the credit expands on the height of stock market bubble but becomes scarce during economic recession when it is most needed. Financial crisis of 2008 reminds us of this pattern of decisions.

4 Conclusions

This paper aimed to investigate the effect of market misevaluation o equity on the outcome of structural model estimates of PD. Merton's model was chosen for modelling the PD due to its simplicity and relatively modest data demands. Probit regression was used to examine the effect of bubble by using proxy of Internet companies default outcomes of 2000. The results suggest that use of bubble-affected stock prices causes Merton's model to understate the probability of default. Throughout the paper we argued that there are very few reasons to believe that our results are due model assumptions other than the equity valuation. On the other hand, there seems to be theoretical ground and large evidence towards a speculative bubble in technology companies during late 90s, the most notable of which are the Internet companies.

Although stock price bubbles have not been proven beyond reasonable doubt, most literature on credit risk that addresses problems (with use of equity markets in modelling risk) cite the Internet bubble as the main candidate. As a suggestion for further research it would be interesting to extend our study to other documented market failures in the US such as housing developments prior to 2008, or perhaps other countries.

Even though we have focused on Merton's model, we believe that our results are applicable to most other structural models that utilize stock market for equity valuation. Only the models that explicitly account for the existence of a bubble can be immune to it. Modelling such complex and random phenomenon has not been met with success so far. This paper will hopefully help to raise awareness of the credit risk research and stir it in the right direction.

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6 Appendix

[1]Basel committee on banking and supervision is an international organisation that has been set us as a branch of the Bank for International Settlements. Its role is to oversee the stability of the banking system by making banks to hold minimum capital requirements. Through this the stability is attained. Its accords BASEL I and BASEL II have been criticized, as the level of requirements is subjective and model dependant. Models that we discuss have been adopted among BASEL member banks and by BASEL.

[*]KMV Corporation is a major developer of credit risk models used in the industry.

[**]CAP stand for Cumulative accuracy Profile and ROC is Receiver operator characteristics; these are common measures of model performance.

[2] For a well presented review of models and their developments see textbook Bilecki and Rutkowski , Springer Finance, 2002

[3]This assumption has proven to be quite unreasonable, and many various debt dynamics models exist today. This however is not affecting our use of Merton's model and doesn't confluence with our aims.

[4]According to EMH and Merton's model assumptions market properly value the firm's equity regardless of economic cycles, economic fundamentals or practically any possible information that can be known to informed investor. Therefore market in theory is forward looking and takes into consideration all that is relevant to determine the appropriated arbitrage-free price. Such arbitrage price is the true price given all the fundamentals and therefore is synonymous with it.

[5] GICS is explained in the data section

[6]more specifically debt has payoff as D minus Put(D) and equity is just a call option with strike equals debt. If debt price is

[7]Please see papers by Bank for International settlement research department on credit risk, namely 'Credit risk measurement and procyclicality ' by Lowe(2002) , also Bharath and Shumway (2004) [pp7] paper.

[8]Under the null hypothesis and in the theory of Black Scholes as well as Meron (1974) setup of the model.

[9]There is another way to solve for the asset parameters. It is known as iterative method. In this method we use past data of equity, debt, risk free, etc. and construct rearranged BS formula for time t-i. This way we can obtain 260 equations of each trading day in a year and solve for parameters from the system. First value of V_A^t is guesses and then the value of d1 and d2 are subsequently put into another BS equation resulting in new V_A^t and the procedure is repeated until convergence. This method should capture some of the dynamics of liability process as it uses more past liability information as our instantaneous solution. The results of PD of our and iterative method differ. However, neither is superior as for some cases iterative procedure is more accurate to ours and for others it is not. Unfortunately we could not apply this method widely as it is too data-demanding.

[11]Please see data section for more information on GICS